Adaptive Scheme for ANOVA Models

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Authors' contributions

This work was carried out in collaboration among all authors. All authors read and approved the final manuscript.

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Abstract

This paper deals with the concept of adaptive scheme and with an application to the Oneway ANOVA model under uncorrelated errors. Oneway ANOVA model is sensitive to nonnormality as well as variance heterogeneity. To overcome these problems, an adaptive scheme is proposed. The adaptive test is a two step procedure. The given data is first examined and classified based on measures of skewness and tailweight. Secondly, a selector statistic is used for selecting a test to be conducted. A 10,000 simulations were conducted to compare the performance of the two models from different continuous distributions. Analysis of real data sets on equal and unequal sample sizes were performed to evaluate the efficiency of the two models. The findings showed that our adaptive scheme outperformed the parametric F-test in symmetric or skewed distributions with varying tailweights except for symmetric and medium-tailed distributions.

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1 Introduction

In this paper, we propose an adaptive scheme for Oneway ANOVA Models under uncorrelated errors. In practtice, often the underlying distribution of a statistical model is not known so a procedure which will maximise power and efficiency should be selected. For example, if the distribution of errors is known to be normal in a linear model then inference based on least squares which maximises power and efficiency should be chosen. On the other hand, when the assumption of normality of the error distribution is violated as a result of outliers, then a more robust methods can be used to analyse the problem. The rank-based method is robust to outliers and has high efficiency and power for both normal and non-normal distributions, see [1] for details. Robustness signifies insensitivity to small deviations from the assumptions of normality [2].

ANOVA model which could be reduced to Gauss Markov model relies mostly on normality, homogeneity of variance and large sample size for it to be modelled. ANOVA models are very sensitive to nonnormality and departures from normality may originate from either skewness or outliers. Furthermore, ANOVA model which are mostly used in clinical trials may have very low enrolment at centres and hence a small sample size. This will inhibit the efficiency of the statistical procedure used.

One particular problem in which normality assumptions become inappropriate is small sample size. In most statistical modelling or techniques, sample size must be large enough for such procedure to be statistically admissible or valid. For small samples, some nonparametric methods have been developed. The advocacy of distribution-free (nonparametric) tests for differences in location problems between samples has been emphasized over the past seven decades [3] .

In this paper, the nine winsorised scores proposed by [4] are considered in the adaptive test of [5]. The adaptive two-sample location problem is extended to the Gauss Markov model (GMM). The Oneway Analysis of Variance (ANOVA) model which is a special case of GMM and the adaptive test are the focus of this paper.

An adaptive test for equality of means in a Oneway layout is described by [6] though he used weighted method. A robust procedure to fit Oneway ANOVA model under adaption on the observed samples was extensively done by [7]. The adaptation in this present study is based on residuals after an initial fit on the observed sample.

1.1 Literature review

In this subsection, the Oneway Analysis of Variance (ANOVA) model which is a special case of Gauss Markov model was briefly reviewed.

1.1.1 Gauss Markov model (GMM)

The Gauss Markov model is given by

$$Y = X\beta + \epsilon \tag{1.1}$$

where Y is an $n \times 1$ vector of observed responses, X is an $n \times p$ (design) matrix of fixed constants, β is a $p \times 1$ vector of fixed but unknown parameters, and ϵ is an $n \times 1$ vector of unobserved random errors. Both Y and ϵ are random vectors. It is assumed that $E(\epsilon) = 0$ and $Cov(\epsilon) = \sigma^2 I$ where σ^2 is some unknown parameter and $\epsilon \sim N(0, \sigma^2 I)$, see [8] and [9].

1.1.2 Oneway analysis of Variance (ANOVA) model

Consider an experiment that is performed to compare $k \geqslant 3$ treatments. For the ith treatment level, suppose that n_i experimental units are selected at random and assigned to the ith treatment.

The Oneway ANOVA model is given by

$$Y_{ij} = \mu + \alpha_i + \epsilon_{ij}, \quad \begin{cases} i = 1, 2, 3, \cdots, k \\ j = 1, 2, 3, \cdots, n_i \end{cases} \qquad (1.2)$$

where the Y_{ij} is the $(ij)^{th}$ observation, μ is a parameter common to all treatments called the overall mean, α_i is the ith treatment effect and random errors ϵ_{ij} are uncorrelated random variables with zero mean and common variance $\sigma^2 > 0$. If the α treatment effects $\alpha_1, \alpha_2, \alpha_3, \cdots, \alpha_k$ are best regarded as fixed constants, then model (1.2) is a special case of model (1.1).
Thus,

$$Y_{n \times 1} = \begin{bmatrix} Y_{11} \\ Y_{12} \\ Y_{13} \\ \vdots \\ Y_{kn_k} \end{bmatrix}, \quad X_{n \times p} = \begin{bmatrix} 1_{n_1} & 1_{n_1} & 0_{n_1} & 0_{n_1} & \cdots & 0_{n_1} \\ 1_{n_2} & 0_{n_2} & 1_{n_2} & 0_{n_2} & \cdots & 0_{n_2} \\ 1_{n_3} & 0_{n_3} & 0_{n_3} & 1_{n_3} & \cdots & 0_{n_3} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 1_{n_k} & 0_{n_k} & 0_{n_k} & 0_{n_k} & \cdots & 1_{n_k} \end{bmatrix}, \quad \beta_{p \times 1} = \begin{bmatrix} \mu \\ \alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_k \end{bmatrix}, \quad \epsilon_{n \times 1} = \begin{bmatrix} \epsilon_{11} \\ \epsilon_{12} \\ \epsilon_{13} \\ \vdots \\ \epsilon_{kn_k} \end{bmatrix}$$

where $p = k + 1$, 1_{ni} is an $n_i \times 1$ vector of ones and 0_n is an $n_i \times 1$ vector of zeros, $E(\epsilon) = 0$, $Cov(\epsilon) = \sigma^2 I$ and

$$n = \sum_{i=1}^{k} n_i$$

see [9].

In Oneway ANOVA model, the test of hypothesis of interest is

$$H_0: \mu_1 = \mu_2 = \cdots = \mu_k$$
$$H_1: \mu_r \neq \mu_s$$

for at least one pair (r, s), $r \neq s$.

Our motivation for the study is to find robust adaptive procedures for parametric tests which are most often used by statisticians and researchers but which are inefficient for nonnormal distributions. In addition, the asymptotic properties of statistical estimates and tests solely rely on the Central Limit Theorem (CLT), however, sample sizes are often not large as in clinical trials.

Real data sets are used to perform a test of hypothesis for Oneway ANOVA models with uncorrelated errors and our proposed scheme in order to ascertain the efficiecies of the two models.

The Introduction of the paper in Section 1 presents the background, the problem and proposed solution,and a brief literature review. The rest of the paper is organised as follows: Section 2 is focused on materials and methods used for the development of the adaptive test. The third section presents the results and discussion of simulation and numerical examples. Section 4 presents the conclusions of the paper highlighting the major findings.

2 Materials and Methods

In this section, the methods and theorem for the development of the adaptive procedures are discussed.

2.1 Rank Test

The rank test considered in this paper is of the form

$$T_\varphi = \sum_{i=1}^{n} \varphi \left[\frac{R(Z_i)}{n+1} \right] I(Z_i = Y_i) \tag{2.1}$$

where Y_i and Z_i are the combined ordered residuals respectively, $a_\varphi(i)$
$= \varphi \left(\frac{i}{n+1} \right)$, $a_\varphi(1), a_\varphi(2), \cdots, a_\varphi(n)$ are scores and φ satisfies the following conditions

- φ nondecreasing function and square-integrable on $(0, 1)$
- φ is differentiable on $(0, 1)$

Thus,

$$\int_0^1 \varphi(u) du = 0$$

and

$$\int_0^1 \varphi^2(u) du = 1,$$

see [10].

2.2 Adaptive test

[11] and [12] distinguished between two different concepts of adaptive procedures; nonrestrictive and restrictive ones. The restrictive procedures are applied in this paper. The Adaptive test of [13] is a two-step procedure. The data is first examined and classified based on measures of skewness and tailweight from a class of continuous distributions. After that a selector statistic is then used for selecting a score for a test to be conducted. This two-staged adaptive procedure maintains the level of significance α for all continuous distribution functions. See [14] for the main theorem behind adaptation.

In this paper, we adapt on residuals, so the combined ordered residuals from an initial fit is used. The measures of skewness and tailweight of the residuals are obtained by using Q_1^* and Q_2^* respectively.

2.3 Selector statistic

The selector statistic $S = (Q_1^*, Q_2^*)$, aids in selecting a score function, where Q_1^* and Q_2^* are the respective measures of skewness and tailweight are defined respectively by

$$Q_1^* = \frac{(m(0.95, 0) - m(0.25, 0.25))}{((m(0.25, 0.25) - m(0, 0.95))} \tag{2.2}$$

$$Q_2^* = \frac{((m(0.95, 0) - m(0, 0.95))}{((m(0.5, 0) - m(0, 0.5))} \tag{2.3}$$

where $m(\alpha_1, \alpha_2) = \frac{1}{h} \sum_{i=t_1+1}^{n-t_2} Z_{(i)}$ and
$Z_i's$ are ordered residuals from an initial fit
$t_1 = [n\alpha_1]$
$t_2 = [n\alpha_2]$
$[x]$ denotes the smallest integer greater than x
$h = n - t_1 - t_2$.

The benchmarks proposed by [15] for the cut off values are used. These benchmarks depend on the sample size n but as $n \to \infty$, the measures converge to those proposed by [5].

For Q_1^*,

$$\text{Lower cut off} = 0.36 + \frac{0.68}{n}$$
$$\text{Upper cut off} = 2.73 - \frac{3.72}{n} \tag{2.4}$$

For Q_2^*,
if $n < 25$,

$$\text{Lower cut-off} = 2.17 - \frac{3.01}{n}$$
$$\text{Upper cut-off} = 2.63 - \frac{3.94}{n} \tag{2.5}$$

if $n \geqslant 25$,
then

$$\text{Lower cut off} = 2.24 - \frac{4.68}{n}$$
$$\text{Upper cut off} = 2.95 - \frac{9.37}{n} \tag{2.6}$$

The cut off points are used to select a rank test which is based on a rank score function corresponding to an unknown distribution. In this paper, the nine Winsorised scores proposed by [4] were considered the most appropriate set of rank scores for testing hypothesis are used. These are classified into four generic scores.

$$\varphi_I(u)) = \begin{cases} s_3, & u > s_1 \\ s_3 + \frac{s_3 - s_2}{s_1}(u - s_1), & otherwise \end{cases}$$

$$\varphi_{II}(u) = \begin{cases} -\frac{s_3}{s_1}(u - s_1), & u < s_1 \\ -\frac{s_4}{s_2 - 1}(u - 1) + s_4, & u > s_2 \\ 0, & otherwise \end{cases}$$

$$\varphi_{III}(u) = \begin{cases} s_2, & u < s_1 \\ s_3 + \frac{s_2 - s_3}{s_1 - 1}(u - 1), & otherwise \end{cases}$$

$$\varphi_{IV}(u) = \begin{cases} s_3, & u < s_1 \\ s_4, & u > s_2 \\ s_3 + \frac{s_4 - s_3}{s_2 - s_1}(u - s_1), & otherwise \end{cases}$$

where s_1, s_2, s_3, s_4 and s_5 are parameters and

$$a_i(j) = \varphi_i\left(\frac{j}{n+1}\right)$$

2.4 Adaptive test and test statistics

Let D_k and φ_k be a region and score selected respectively as in Fig. 1, with $k = 1, 2, ..., 9$, then the adaptive test, $AD(S, \varphi)$, is

$$AD(S, \varphi) = T_{\varphi_k}, S \in D_k \tag{2.7}$$

where

$$T_{\varphi_k}(\Delta) = \sum_{i=1}^{n_2} a_{\varphi_k}[R(y_i - \Delta)] \tag{2.8}$$

is a test statistic based on the ranks and score φ_k associated with region D_k and hence distribution-free. Under H_0, the mean of $T_{\varphi_k}(\Delta)$ is zero.

The nine regions which depend on the selector statistics $S = (Q_1^*, Q_2^*)$ are defined by:

$D_1 = LH = Q_1^* < \hat{Q}_{1l}^*, Q_2^* > \hat{Q}_{2u}^*$
$D_2 = SH = \hat{Q}_{1l}^* < Q_1^* < \hat{Q}_{1u}^*, Q_2^* > \hat{Q}_{2u}^*$
$D_3 = RH = Q_1^* > \hat{Q}_{1u}^*, Q_2^* > \hat{Q}_{2u}^*$
$D_4 = LM = Q_1^* < \hat{Q}_{1l}^*, \hat{Q}_{2l}^* < Q_2^* < \hat{Q}_{2u}^*$
$D_5 = SM = \hat{Q}_{1l}^* < Q_1^* < \hat{Q}_{1u}^*, \hat{Q}_{2l}^* < Q_2^* < \hat{Q}_{2u}^*$
$D_6 = RM = Q_1^* > \hat{Q}_{1u}^*, \hat{Q}_{2l}^* < Q_2^* < \hat{Q}_{2u}^*$
$D_7 = LL = Q_1^* < \hat{Q}_{1l}^*, Q_2^* < \hat{Q}_{2l}^*$
$D_8 = SL = \hat{Q}_{1l}^* < Q_1^* < \hat{Q}_{1u}^*, Q_2^* < \hat{Q}_{2l}^*$
$D_9 = RL = Q_1^* > \hat{Q}_{1u}^*, Q_2^* < \hat{Q}_{21}^*$

where $\hat{Q}_{1l}^*, \hat{Q}_{1u}^*, \hat{Q}_{2l}^*, \hat{Q}_{2u}^*$ are benchmarks from the ordered residuals, see [15].
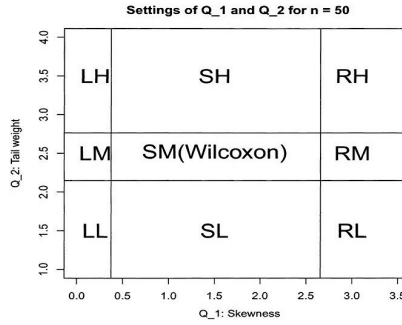

Fig. 1. Plot of scores with n = 50

2.5 Proposed adaptive scheme

The procedure for the proposed adaptive scheme is as follows:

1. Let $Y_{(1)}, Y_{(2)} \ldots, Y_{(N)}$ be the combined ordered residuals of $Y_{11}, Y_{12}, \ldots, Y_{1n_1}, Y_{21}, Y_{22}, \ldots, Y_{2n_2}$, $Y_{n1}, Y_{n2}, \ldots, Y_{nn_k}$ from continuous distribution function $f(t)$ with some amount of variations denoted by Δ among the residuals, that is $f(t - \Delta)$.

2. The unknown dstribution of the residuals will be classified using both the selector statistic $S = (Q_1^*, Q_2^*)$ and the cut-off values.

3. Once the distribution of these residuals is classified, a corresponding score of the unknown distribution is selected.

4. After the selection of the score function, the test is then conducted.

Hypothesis Testing
We wish to test

$$H_0 : \Delta = 0$$
$$H_1 : \Delta \neq 0$$

3 Results and Discussion

This secction presents the findings of the study of a 10,000 simulations and numerical examples.

3.1 Simulation results

A 10,000 simulations were conducted for Normal (Norm), Contaminated Normal (CNorm), Logistic (Logis), Laplace (Lap), Lognormal (LNorm), Exponential (Exp), Cauchy (Cau), Weibull (Wei), Mixture (Mixt) and Pareto (Par) distributions.

3.1.1 Classification Table

Table 1 displays the classification of performance of $S = (Q_1^*, Q_2^*)$ for sample size $n_1 = n_2 = n_3 = 100$ for each of the distributions.

Table 1. Skewness and Tailweight classification of adaptive test

Sel Stats	Norm	CNorm	Logis	Lap	LNorm	Exp	Cau	Wei	Mixt	Par
LH	0	0	0	0	0	0	1702	0	1507	0
LL	0	0	0	0	0	0	0	0	0	0
LM	0	0	0	0	0	0	0	0	0	0
RH	0	0	0	0	9939	2958	1508	2939	1409	10000
RL	0	0	0	0	0	0	0	0	0	0
RM	0	0	0	0	61	7042	0	7058	0	0
SH	5	9	2548	9898	0	0	6790	0	7083	0
SL	1	1	0	0	0	0	0	0	0	0
SM	9994	9990	7452	102	0	0	0	3	1	0

Source: Simulation study

The Normal, Contaminated Normal, Logistic, Laplace, Cauchy and Mixture of distributions were all symmetric but have different tailweights. The Lognormal and Pareto distributions were classified as right-skewed and heavy-tailed. The Exponential and Weibull distributions were also identified as right- skewed and medium-tailed distributions.

The normal and contaminated normal distributions were correctly classified to be symmetric and medium-tailed by 99.94% and 99.90% respectively. The Laplace distribution is correctly classified as a symmetric and heavy-tailed by 98.98% and Lognormal distribution as a right-skewed and heavy-tailed by 99.39%. The Pareto distribution on the other hand was 100% classified as a right-skewed and heavy-tailed distribution.

A simulation was conducted to compare the parametric F-test and the adaptive test. For the parametric, F test statistic (Value) and the residual standard error (σ) were obtained. In the case of the adaptive test, the underlying error distribution (score), test statistic (value), scale parameters for the sample and residual (τ_s) and (τ_r) respectively were obtained. The comparison is presented on Tables 2, 3, 4, 5 and 6.

3.1.2 Normal distribution

Using the normal distribution with the location parameter $\mu = 0$ and scale parameter $\sigma = 1$, under H_0, 10,000 simulations were run for the various sample sizes.
Table 2 displays the simulation results for the Normal distribution. The selector statistic for the adaptive test identified the normal distribution as a symmetric and medium-tailed distribution. The parametric F-test performed better than the adaptive test at all the sample sizes considered. However, at 5% or 1% significance level, the two models failed to reject the null hypothesis of no difference in level means.

Table 2. Normal distribution

Sample Size	F-Test			Adaptive Test		
(n_1, n_2, n_3)	Value	σ	Score	Value	τ_s	τ_r
$(5, 5, 5)$	1.1716	0.9831	SH	2.2919	1.0601	1.0266
$(10, 10, 10)$	1.0788	0.9901	SM	2.1401	1.0297	1.0201
$(15, 15, 15)$	1.0507	0.9935	SM	2.1022	1.0274	1.0231
$(20, 20, 20)$	1.0367	0.9943	SM	2.0485	1.0302	1.0288
$(25, 25, 25)$	1.0237	0.9957	SM	2.0286	1.0307	1.0304
$(30, 30, 30)$	1.0138	0.9974	SM	2.0024	1.0292	1.0294
$(50, 50, 50)$	1.0049	0.9981	SM	2.0023	1.0293	1.0294
$(100, 100, 100)$	1.0201	0.9993	SM	2.0269	1.0271	1.0271

Source: Simulation study

3.1.3 Laplace distribution

The results for 10,000 simulations carried out for Laplace distribution with the location parameter $\mu = 0$ and a scale parameter $\lambda = 1$ is displayed on Table 3.

Table 3. Laplace distribution

Sample Size	F-Test			Adaptive Test		
(n_1, n_2, n_3)	Value	σ	Score	Value	τ_s	τ_r
$(5, 5, 5)$	1.1641	1.3535	SH	2.3008	1.2634	1.2377
$(10, 10, 10)$	1.0680	1.3856	SH	2.1494	1.1955	1.1915
$(15, 15, 15)$	1.0593	1.3958	SH	2.1568	1.1501	1.1496
$(20, 20, 20)$	1.0445	1.3988	SH	2.1604	1.1276	1.1270
$(25, 25, 25)$	1.0177	1.4048	SH	2.0619	1.1308	1.1305
$(30, 30, 30)$	1.0383	1.4050	SH	2.0784	1.1300	1.1301
$(50, 50, 50)$	1.0250	1.4063	SH	2.0568	1.1121	1.1120
$(100, 100, 100)$	1.0112	1.4112	SH	2.0220	1.0956	1.0955

Source: Simulation study

The selector statistic for the adaptive test classified the Laplace distribution as a symmetric and heavy-tailed distribution. The adaptive test outperformed the F-test as the variance returned for the adaptive test is smaller than the F-test. At 5% or 1% level of significance, the two tests failed to reject the null hypothesis.

3.1.4 Cauchy distribution

The Cauchy distribution with the location parameter $\mu = 0$ and a scale parameter $\gamma = 1$ is simulated 10,000 under H_0. The results of the simulation is shown on Table 4.

The Cauchy distribution is identified by the selector statistic as a symmetric and heavy-tailed distribution. The adaptive test outperformed the traditional F-test as the variance returned by the adaptive test is smaller than the F-test. Both tests however, failed to reject the null hypothesis of no difference in level means at 5% or 1% significance level.

3.1.5 Weibull distribution

Simulation Results for Weibull distribution with the shape parameter $k > 0$ and a scale parameter $\lambda = 1$ is shown on Table 5.

Table 4. Cauchy distribution

Sample Size	F-Test		Adaptive Test			
(n_1, n_2, n_3)	Value	σ	Score	Value	τ_s	τ_r
$(5, 5, 5)$	1.0899	26.2994	LH	2.0829	2.7886	2.7844
$(10, 10, 10)$	1.0320	39.1821	LH	2.2975	2.3935	2.4009
$(15, 15, 15)$	1.0158	44.0567	SH	2.1192	1.9799	1.9824
$(20, 20, 20)$	1.0171	64.7010	SH	2.0545	1.9060	1.9073
$(25, 25, 25)$	1.0109	61.5071	SH	2.0691	1.8858	1.8865
$(30, 30, 30)$	1.0104	76.9902	SH	2.0246	1.9384	1.9386
$(50, 50, 50)$	1.0076	64.0498	SH	2.0381	1.8584	1.8588
$(100, 100, 100)$	1.0076	110.0536	SH	2.0110	1.7335	1.7339

Source: Simulation Study

Table 5. Weibull distribution

Sample Size	F-Test		Adaptive Test			
(n_1, n_2, n_3)	Value	σ	Score	Value	τ_s	τ_r
$(5, 5, 5)$	1.1757	0.9423	RH	2.0644	0.7075	0.7002
$(10, 10, 10)$	1.0605	0.9705	RM	1.7301	0.6291	0.6486
$(15, 5, 15)$	1.0598	0.9788	RM	2.0527	0.5945	0.6121
$(20, 20, 20)$	1.0154	0.9834	RM	2.0755	0.5643	0.5665
$(25, 25, 25)$	1.0306	0.9840	RM	2.0731	0.5641	0.5689
$(30, 30, 30)$	1.0252	0.9870	RM	1.9691	0.5590	0.5692
$(50, 50, 50)$	1.0183	0.9926	RM	2.0214	0.5530	0.5542
$(100, 100, 100)$	0.9953	0.9961	RM	1.9813	0.5252	0.5225

Source: Simulation study

The Weibull distribution was identified by the selector statistic as a right-skewed and medium-tailed distribution. The adaptive test outperformed the F-test. It is worth noting that as the sample size increases the residual standard error (σ) increases but the estimated scale parameters (τ_s) and (τ_r) decrease. Notwithstanding, both tests failed to reject the null hypothesis at 5% or 1% level of significance.

3.1.6 Mixture of distributions

A 10,000 simulations of mixture of distributions for three samples were generated from the Normal $N(0, 1)$, Laplace ($\mu = 0, \lambda = 1$) and Cauchy ($\mu = 0, \gamma = 1$) distributions respectively and is shown on Table 6.

Table 6. Mixture of distributions

Sample Size	F-Test		Adaptive Test			
(n_1, n_2, n_3)	Value	σ	Score	Value	τ_s	τ_r
$(5, 5, 5)$	1.1740	10.6971	SH	2.6740	1.6893	1.7001
$(10, 10, 10)$	1.0934	10.0139	LH	2.6780	1.6295	1.6261
$(15, 15, 15)$	1.0369	88.1010	SH	2.6322	1.5095	1.5088
$(20, 20, 20)$	1.0345	14.2196	SH	2.6094	1.4928	1.4922
$(25, 25, 25)$	1.0229	14.5802	SH	2.6327	1.5079	1.5082
$(30, 30, 30)$	1.0280	16.3163	SH	2.5979	1.5414	1.5415
$(50, 50, 50)$	1.0151	32.4284	SH	2.7024	1.5213	1.5211
$(100, 100, 100)$	0.9995	39.4103	SH	3.0333	1.4866	1.4865

Source: Simulation study

The mixture of distributions was classified by the selector statistic as a symmetric and heavy-tailed distribution. The variance returned suggest that the adaptive test outperformed the F-test at all levels of the sample sizes. However, the two tests failed to reject the null hypothesis of no difference in level means at 5% or 1% significance level.

3.2   Numerical examples

3.2.1   Example for equal sample size

Three different pain relief drugs were administered on 27 patients suffering from migraine headache. Table 7 shows time in minutes of relief from the migraine headache. This is an extract from [16].

Table 7. Time of relief for migraine headache sufferers

Drug A	4	5	4	3	2	4	3	4	4
Drug B	6	8	4	5	4	6	5	8	6
Drug C	6	7	6	6	7	5	6	5	5

From Fig. 2, the assumption of normality to the data on Table 7 is not appropriate. An outlier in Drug B is obvious. The result of the analysis is displayed on Table 8.
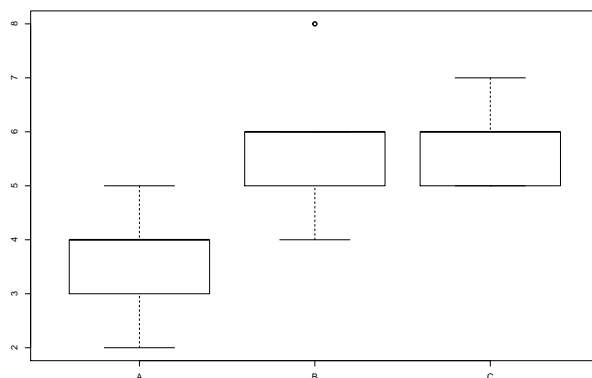

Fig. 2. Time of relief for migraine headache sufferers

Table 8. Time of relief for migraine headache sufferers

Sample Size	F-Test			Adaptive Test			
(n_1, n_2, n_3)	Value	p-value	σ	Score	Value	p-value	τ
$(9, 9, 9)$	11.91	0.0003	1.089	SH	13.6881	0.0001	0.8788

Source: Table 7

The selector statistic classified the underlying error distribution of the data as a symmetric and heavy-tailed distribution. The adaptive test reported the least variance, that is, the adaptive test is more efficient than the F-test. However, both models rejected the null hypothesis, H_0 at 5% or 1% significance level. Thus, there are differences in mean time for the pain relief. The asymptotic relative efficiency (ARE) of the adaptive test over the F-test is about 65.1%.

3.2.2 Example for unequal sample sizes

Four catalyts that may affect the concentration of one component in a three-component liquid mixture are being investigated. The data as shown on Table 9 is extracted from [17].

Table 9. Catalyst

1	2	3	4
58.2	56.3	50.1	52.9
65.2	54.5	54.2	49.9
58.4	57.0	55.4	50.0
55.8	55.3		51.7
54.9			

The results of the analysis is presented on Table 10.

Table 10. Catalyst

Sample Size	F-Test			Adaptive Test			
(n_1, n_2, n_3, n_4)	Value	p-value	σ	Score	Value	p-value	τ
$(5, 4, 3, 4)$	5.8778	0.0104	2.7472	SL	7.9989	0.0056	1.9989

Source: Table 9

The selector statistic classified the underlying error distribution of the data as a symmetric and light-tailed distribution. The adaptive test performed better than the F- test because the variance returned ($\sigma = 2.7472$) is greater than the scale parameter ($\tau = 1.9989$). At 5% or 1% significance level, both tests rejected the null hypothesis. As a consequence, the four catalysts do not have the same mean effect on the concentration of the three-component liquid mixture. The asymptotic relative efficiency of the adaptive test over the F-test is about 52.95%.

4 Conclusions

In this paper, the Oneway Analysis of Variance (ANOVA) model was compared with the adaptive test. We used the two dimensional selector statistic $S = (Q_1^*, Q_2^*)$, where Q_1^* and Q_2^* are respective measures of skewness and tailweight of the unknown distribution function. The usage of the nine winsorised scores accommodated a wide range of distributions which are either symmetric or asymmetric with varying tailweights as shown on Table 1.

The performance of these tests at small sample sizes was of much interest in this study because most sensitive areas of the application of ANOVA models such as cinical trials often has low enrolment. The adaptive test was more efficient than the traditional ANOVA F-test at very small sample sizes, in the presence of outliers and nonnormal distributions. As a consequence, the adaptive test should be taken note of in statistical modelling.

Competing Interests

Authors have declared that no competing interests exist.

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