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# Many Kinds of Reserved Judgement in No-Data Problem

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## Abstract

The reserved judgment can be broadly categorized into three types: Re-Do, Re-Set, and Natural Flowing Case (*i.e.* step by step in Re-Try). Hori *et al.* constructed the Bayes-Fuzzy Estimation and demonstrated that system theory can be applied to the possibility of Markov processes, and that decision-making approaches can be applied to sequential Bayes estimation. In this paper, we focus on the Natural Flowing Case within reserved judgment. Here, the possibility of oblique (or principal) factor rotation is considered as a part of the tandem fuzzy system that follows step by step for sequential Bayes estimation. Ultimately, we achieve a significant result whereby the expected utility can be calculated automatically without the need to construct a utility function for reserved judgment. There, this utility in Re-Do can be calculated by the prior utility, and that utility in Re-set does not exist by our research in this paper. Finally, we elucidate the relationship between fuzzy system theory and fuzzy decision theory through an applied example of Bayes-Fuzzy theory. Fuzzy estimation can be applied to only normal making decision, but it is impossible to apply abnormal decision problem. Our Vague, specially Type 2 Vague can be applied to abnormal case, too.

## Keywords

Bayes-Fuzzy Estimation, Re-Do, Re-Set, Indifferent Zone, Reserved Judgement

## 1. Introduction

Zadeh has defined fuzzy sets and the probability of fuzzy events [1]. This definition requires the condition of the direct sum of the possibility distribution of fuzzy events (considered as mutually exclusive events in fuzzy systems theory), which is reflected in the orthogonal condition of the possibility distribution.

Subsequently, Okuda *et al.* constructed decision-making methods for fuzzy events in ambiguous environments and established fuzzy Bayesian inference [2]. Following this, Uemura (Hori) focused on the extension principle of Zadeh's mappings to construct Decision Making in Type 1 Vague Events [3] and Decision Making in Type 2 Vague Events [4], thereby establishing Bayesian fuzzy inference. Finally, Hori *et al.* demonstrated that systemic examples of Bayesian fuzzy inference could be represented by Markov processes, while deterministic examples were characterized by sequential Bayesian inference [4]. It is noted that when fuzzy events are additive, the transition matrix of the possibility Markov process corresponds to the possibility principal factor rotation [5], and when they are non-additive, the matrix represents a possibility oblique factor rotation [6].

In this paper, we address the issue of judgment reservation within the context of no-data problems [7]. Judgment reservation is broadly categorized into re-attempt (Re-Do), re-setting (Re-Set), and general cases (Re-Try) (not artificial). Here, the re-attempt and re-setting are decision-making issues that are artificially concluded in two stages. It is important to note that utility functions within these judgment reservations do not require prior setting; they are automatically suggested by the serial-type fuzzy system that performs the calculation.

## 2. Probability Distribution of Indiscriminate Events

In a conventional possibility space, denoted as ($S$, $K$, $\Pi$), $S$ represents the states of nature, $K$ denotes a σ-algebra comprising subsets of $S$, and $\Pi$ signifies a possibility measure. Herein, a fuzzy set, referred to as a fuzzy event $F$, is characterized by a measurable possibility distribution $\Pi_F(S): S \to (0,1)$ over $S$. In this paper, we assume that possibility distributions $\Pi_{FK}(K = 1,\cdots,n)$ for two or more non-direct sum fuzzy events have been predetermined by the decision-maker.

### 2.1. In the Case of a Retry (Re-Do)

In this section, we consider the case where $\sum_{k=1}^{n} \Pi_{Fk}(S) \le 1 \quad \forall_S \in S$. Here, to circumvent the risks associated with decision-making due to insufficient information content of fuzzy events, we introduce the concept of the event of indifference, denoted as $F_e$. The probability distribution of this event of indifference can be automatically derived using the following equation.

$$\Pi_{Fe}(S) = 1 - \sum_k \Pi_{Fk}(S) \tag{1}$$

In this paper, two fuzzy events are denoted as $N \triangleq F_1$ and $M \triangleq F_2$, while the event of indifference is designated as $F_e$. Here, we exemplify by considering the case of a binary choice for an entrance examination recommendation, which exemplifies a no-data problem. The natural state $S$ is assumed to range from 0 to 100 points based on internal assessment scores. Now, let $M$ = {good internal assessment score} and $N$ = {poor internal assessment score} be set by the decision-maker as depicted in **Figure 1**. From Equation (1), the possibility distribution for the event of indifference, *Fe*, is automatically derived.

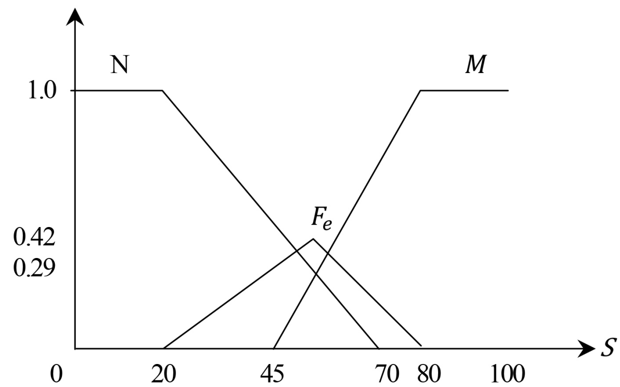


**Figure 1.** Indiscriminate events (Re-Do).

The examination presents a no-data issue, where prior information (school records) is available, yet observational information (from the written examination) is lacking. Through a judgment postponement (in this case, an interview examination) (Re-Do), observational information can be newly obtained, reducing the second stage to a typical statistical decision-making problem. It is important to note that this decision-making problem forcibly halts sequential Bayesian inference after two stages.

In this section, we interpret the indiscriminate event $F_e$ by categorizing it according to zones on the state of nature (refer to **Figure 1**). In Zone X where $0 \le s < 20$, the event is entirely $N$. In Zone a where $20 \le s < 45$, it is a conditioned indiscriminate event known to be a fuzzy event $N$. Zone b, defined by $45 \le s < 70$, represents an indiscriminate event that is neither fuzzy event $N$ nor $M$, although the relationship in magnitude between fuzzy events $N$ and $M$ is known. In Zone c where $70 \le s < 80$, the event is a conditioned indiscriminate event understood to be fuzzy event $M$. Finally, in Zone Y where $80 \le s \le 100$, the event is completely $M$.

Furthermore, as this is a binary choice problem, the decisions are set as $D_1$ = {Pass} and $D_2$ = {Fail}. To mitigate the risk associated with the lack of information due to the introduction of indiscriminate events, $D_3$ = {Retest(Re-Do)} is added. In this section, the decision-maker is considered to have a risk-neutral utility function $U\left(S|D\right)$, which is determined by the certainty equivalent method via lottery [8] (**Figure 2**).

## 2.2. In the Case of Reconfiguration (Re-Set)

In the previous section, rather than conducting interviews as remedial examinations (Re-Do), there are instances where it has been adopted to retake the examination entirely (Re-Set). This problem of decision-making often arises due to issues on the part of the admitting body, such as errors in question setting. It also poses a potential risk of leading to the admission of all examinees, which makes it an exceedingly perilous decision-making problem. Consequently, this decision-making issue is considered a Max-Max problem.

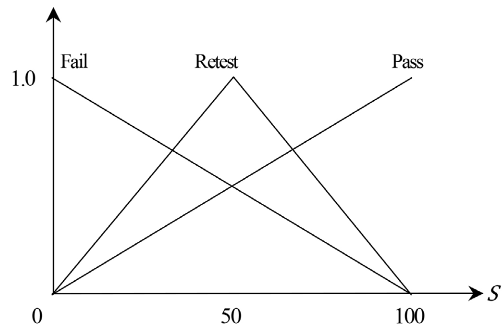


**Figure 2.** Utility function (Risk-Neutral).

In the context discussed herein, the probability distribution of the indeterminate event is computed automatically using the Three-Point Estimation method (20, 50, 80), yielding the probability distribution L-L (50, 30) (refer to **Figure 1**). Subsequently, the probability distributions for the fuzzy events M and N are recalibrated to achieve direct summation with the probability distribution of the indeterminate event.

## 3. Decision-Making Methods in Re-Do (Retry Attempts)

The representative value grade of the possibility distribution in indiscriminate events is always less than one. Consequently, in accordance with conventional statistical decision-making methods, decisions are made based on the principle of expected utility maximization, which involves an integral transformation of the possibility distribution and the utility function [7].

$$\begin{aligned} E(D_1) &= \int \pi(S) \times \mu_N\left(U_1^{-1}(S \mid D_1)\right) \mathrm{d}S \\ E(D_2) &= \int \pi(S) \times \mu_M\left(U_2^{-1}(S \mid D_2)\right) \mathrm{d}S \\ E(D_3) &= \int \pi(S) \times \mu_{Fe}\left(U_3^{-1}(S \mid D_3)\right) \mathrm{d}S \end{aligned} \tag{2}$$

In the event that a deferred judgment becomes the optimal course of action, a two-stage sequential Bayesian inference is employed. For the sake of simplicity, the following explanation utilizes the example of the recommendation test from the previous section. Utilizing the expected feasibility measure weighted by the likelihood information quantity as defined in Equation (3), we propose a two-stage feasibility state intention decision-making method.

$$H_i = \max\nolimits_S \prod\nolimits_{Fi}(S) \cdot \log \prod\nolimits_{Fi}(S) \tag{3}$$

**(Step 1) Unidimensional Decision-Making Method**

$H_1E(D_1) + H_2E(D_2) \ge H_3E(D_3)$ then the optimal decision is $E(D_1) \ge E(D_2)$. Conversely, if $D^* = D_1$ $E(D_1) < E(D_2)$, the optimal decision is $D^* = D_2$.

Decision-making concluded.

When the condition $H_1E(D_1) + H_2E(D_2) < H_3E(D_3)$ holds, let $D^* = D_3$.

Proceed to Step 2.

**(Step 2) Two-Stage Decision-Making Method (In Case of Retrial)**

When $H_1E(D_1)+H_2E(D_2)<H_3E(D_3)$

1) When $H_3=\max_i H_i\,(i=1,2,3)$, let us define the following transformations:

$$H_1' \triangleq H_3 - H_1$$

$$H_2' \triangleq H_3 - H_2$$

$$H_3' \triangleq \max(H_1', H_2')$$

Subsequently, replace $H_i$ with $H_i'$ and return to Step 1.

2) Other Considerations:

When $H_1E(D_1)\geq H_2E(D_2)$ then, $D^*=D_1$

When $H_1E(D_1)<H_2E(D_2)$ then, $D^*=D_2$

Decision-making concluded.

## 4. Decision-Making in Re-Setting (Reconfiguration)

Initially, as a measure of the relationship between magnitude in terms of possibility and necessity of fuzzy numbers, Equations (4), (5), (6), and (7) have been defined [9].

Subsequently, as an example of the direct sum fuzzy system theory in Bayesian fuzzy reasoning, a possibility Markov process with potential principal factor rotation represented by transition matrices $M_1$ and $M_2$ has been derived [5]. The decision-making problem of Re-Setting (reconfiguration) transforms into a Max-Max problem that does not take deferral of judgment into account, due to the post-reconfiguration decomposition of the direct sum fuzzy system into two partitions. Therefore, the Max-Product method, corresponding to the risk-tolerant decision-making problems described by Tanaka *et al.*, is adopted [10]. Due to space constraints, this paper will avoid complex theoretical development and will present qualitative results that disregard deferral of judgment for the sake of clarity.

$$POS(M\geq N)\triangleq \sup_{U\geq V}\min\left(\mu_M(U),\mu_N(V)\right) \tag{4}$$

$$POS(M> N)\triangleq \sup_{U}\inf_{V\geq U}\min\left(\mu_M(U),\mu_N(V)\right) \tag{5}$$

$$NES(M\geq N)\triangleq \inf_{U}\sup_{V\leq U}\max\left(1-\mu_M(U),\mu_N(V)\right) \tag{6}$$

$$NES(M> N)\triangleq 11-\sup_{U\geq V}\min\left(\mu_M(U),\mu_N(V)\right) \tag{7}$$

1) $20\leq S\leq 50$ (**Figure 3**) Possibility Principal Factor Rotation

$$M_1=\begin{bmatrix} POS(N\geq Fe) & NES(N>Fe) \\ NES(N>Fe) & POS(N\geq Fe)\end{bmatrix} \tag{8}$$

2) $50\leq S\leq 80$ (**Figure 3**) Possibility Principal Factor Rotation

$$M_2=\begin{bmatrix} POS(M\geq Fe) & NES(M>Fe) \\ NES(M>Fe) & POS(M\geq Fe)\end{bmatrix} \tag{9}$$

3) Max Product Method

$$\Pi(D_1)=\max_s \pi(S)\times\mu_N\left(U_1^{-1}(S\mid D_1)\right) \tag{10}$$

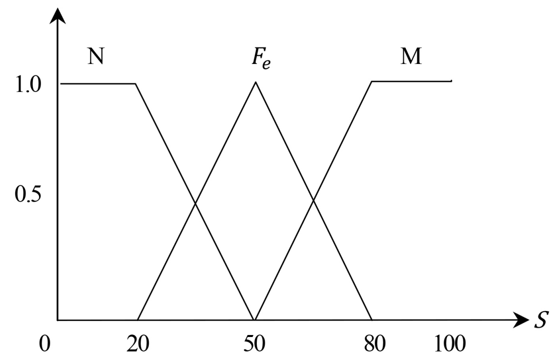


**Figure 3.** Indiscriminate events (Re-Set).

$$\Pi(D_2) = \max_s \pi(S) \times \mu_M\left(U_2^{-1}(S \mid D_2)\right) \tag{11}$$

Equations (4) and (7) yield that the sum of the row and column elements for $M_1$ and $M_2$ becomes 1 given that $POS(M \geq N) + NES(M > N) = 1$. By sequentially inputting/outputting the initial inputs $\left[\Pi(D_1), \Pi(D_2)\right]^{\mathrm{T}}$ through the two-stage cascaded fuzzy systems $M_1$ and $M_2$, the possibility fluctuation rate (area centroid) remains constant. The final output will invariably fall below the expected possibility utility as dictated by Equations (10) and (11). The ambiguity represented by the state of indecision is eliminated through the process of two successive direct sum possibility principal factor rotations. This equates to one rotation of a possibility principal factor in fuzzy system theory, which is equivalent to two rotations of a varimax rotation in multivariate analysis [6], and the second rotation of the possibility principal factor completes a full 360-degree turn. Consequently, the state of indecision is disregarded, and the decision carrying the maximum expected possibility utility, as described by Equations (10) and (11), becomes the optimal action in a conventional decision-making problem. After all, this decision problem has not the reserved judgement, *i.e.* go-to-not problem.

## 5. Non-Arbitrary (Natural) Decision Pending

Firstly, divergent from the case of Re-Do in Chapter 3, decisions are made after the fuzzy system is decomposed in each zone. Subsequently, a distinction from the Re-Set scenario in Chapter 4 is that since the indiscriminate and existing fuzzy events are not direct sums, a four-stage series-type fuzzy system—$MM_1$, $MM_2$, $MM_3$, and $MM_4$—must be constituted based on the possibility skew orthogonal factor rotation. It is noteworthy in this decision-making issue that the representative grade of the indiscriminate event always remains less than 1, hence, the fuzzy system is quartered and reassembled, which necessitates adopting the Min-Max principle.

1) Potential Oblique Factor Rotation Matrix in Zone A

$$MM_1 = \begin{bmatrix} POS(N \geq Fe) & NES(N > Fe) \\ NES(N > Fe) & POS(N \geq Fe) \end{bmatrix} \tag{12}$$

2) Potential Oblique Factor Rotation Matrix in Zone B

$$MM_2 = \begin{bmatrix} POS(Fe \geq N) & NES(Fe > M) \\ NES(Fe > M) & POS(Fe \geq N) \end{bmatrix} \tag{13}$$

3) Potential Oblique Factor Rotation Matrix in Zone C

$$MM_3 = \begin{bmatrix} POS(Fe \geq M) & NES(Fe > N) \\ NES(Fe > N) & POS(Fe \geq M) \end{bmatrix} \tag{14}$$

4) Potential Oblique Factor Rotation Matrix in Zone D

$$MM_4 = \begin{bmatrix} POS(M \geq Fe) & NES(M > Fe) \\ NES(M > Fe) & POS(M \geq Fe) \end{bmatrix} \tag{15}$$

5) Min-Max Principle

$$\wedge(D_1) = \min_S \max\left(\pi(S), \mu_N\left(U_1^{-1}(S \mid D_1)\right)\right) \tag{16}$$

$$\wedge(D_2) = \min_S \max\left(\pi(S), \mu_M\left(U_2^{-1}(S \mid D_2)\right)\right) \tag{17}$$

Upon providing the initial input T with conjunctions $\left[\wedge(D_1), \wedge(D_2)\right]^{\mathrm{T}}$, the input and output to the four-stage series fuzzy systems $MM_1$, $MM_2$, $MM_3$, and $MM_4$ are sequentially repeated. Subsequently, the possibility information quantity as expressed in Equation (3) and the final output are combined through a weighted sum to calculate the expected utility of deferring judgment. Although this process yields two expected utilities of deferring judgment after the weighted sum, the average of these, adopted in a risk-neutral manner, is employed. Decision-making involves determining the optimal action by identifying and selecting the decision associated with the maximum value among the expected utility of deferring judgment, as outlined in Equations (16) and (17). After all, we consider that Fuzzy-Bayes estimation can not apply to this decision problem, however, Bayes-Fussy estimation is adaptive for solving that problem.

## 6. Conclusions

In the initial case of reattempt (Re-Do), we constructed a decision-making method that, while rational, involves a semi-artificial manipulation to ensure that the expected utility of postponing the judgment is minimized upon second consideration.

Subsequently, in the case of resetting (Re-Set), it is demonstrated that the problem becomes a Max-Max issue, thus normal decision-making that disregards judgment postponement is adopted.

Finally, regarding general judgment postponement, Equations (13) and (14) do not result in row and column sums equal to one. This is due to the ambiguity inherent in the potential non-orthogonal sum of fuzzy events, which indicates that even after four rotations, there is a possibility of falling into judgment postponement. As a future issue, after experiencing judgment postponement following four rotations, we aim to reconstruct a sequential Bayesian-fuzzy decision theory for the next decision-making process by revisiting the reattempt or reset options from the perspectives of possibility information quantity and possibility volatility rate (area centroid).

Fuzzy-Bayes estimation can be complete included in Bayes estimation. But, my proposed estimation for reserved judgement is called as type 2 Fuzzy-Bayes in which exists an example for Bayes. On the other hand, Type 1 Bayes-Fuzzy estimation can be mapping from type 1 vague events in other world to type 1 fuzzy events in this world. Furthermore, type 2 Bayes-Fuzzy can be mapping from type 2 vague events in other world around another world to type 2 fuzzy events in this world. Note that type 1 vague events are normal case (ex. under peace) and type 2 vague events are abnormal case (ex. under war). Expect of abnormal decision problem, we must pick up the bayes estimation. But, in abnormal decision problem (ex. war), we must choice type 2 Vague estimation.

## Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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