Hindawi
Advances in Mathematical Physics
Volume 2022, Article ID 5969042, 5 pages
https://doi.org/10.1155/2022/5969042

Research Article

Dynamical Property of the Shift Map under Group Action

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Received 14 December 2021; Revised 11 May 2022; Accepted 14 December 2022; Published 22 December 2022

Academic Editor: Mohammad Mirzazadeh

Firstly, we introduced the concept of G-Lipschitz tracking property, G-asymptotic average tracking property, and G-periodic tracking property. Secondly, we studied their dynamical properties and topological structure and obtained the following conclusions: (1) let (X, d) be compact metric G-space and the metric d be invariant to G. Then, σ has $\bar{G}$-asymptotic average tracking property; (2) let (X, d) be compact metric G-space and the metric d be invariant to G. Then, σ has $\bar{G}$-Lipschitz tracking property; (3) let (X, d) be compact metric G-space and the metric d be invariant to G. Then, σ has $\bar{G}$-periodic tracking property. The above results make up for the lack of theory of G-Lipschitz tracking property, G-asymptotic average tracking property, and G-periodic tracking property in infinite product space under group action.

1. Introduction

At present, shadowing property has gradually become an important theory and concept in dynamical system. Relevant results are seen in [1–11]. For example, Wang and Zeng [1] proved that if f has q-average tracking property, then f is chain transitivity under some conditions. Wu [2] showed that f has tracking property and σ has tracking property are equivalent. Ji et al. [3] proved that $f \circ g$ has the Lipschitz shadowing property and $\sigma_{f \circ g}$ has the Lipschitz shadowing property are equivalent in the double inverse limit space. Ahmadi and Hosseini [4] gave that δ-ergodic pseudo-orbit of a system means chain mixing. Fakhari and Ghane [5] introduced some kind of specification property. Niu [6] showed that the average-shadowing property and dense minimal set means weakly mixing. Oprocha et al. [7] gave equivalent conditions for shadowing. Hossein and Reza [8] investigated the relations of various shadowing. Pierre and Thibault [9] studied shadowing and periodic shadowing properties.

Firstly, we introduced the concept of G-Lipschitz tracking property, G-asymptotic average tracking property, and G-periodic tracking property. Secondly, we studied their dynamical properties and topological structure and obtained the following conclusions:

Theorem 1. *Let (X, d) be compact metric G-space and the metric d be invariant to G. Then, σ has $\bar{G}$-asymptotic average tracking property.*

Theorem 2. *Let (X, d) be compact metric G-space and the metric d be invariant to G. Then, σ has $\bar{G}$-Lipschitz tracking property.*

Theorem 3. *Let (X, d) be compact metric G-space and the metric d be invariant to G. Then, σ has $\bar{G}$-periodic tracking property.*

2. $\bar{G}$-Asymptotic Average Shadowing Property of σ

The concept of metric G-space is shown in [12]. We can find the concept that d is invariant to G in [13]. The concepts of zero density set and G-asymptotic average tracking property are seen in [14, 15]. The concept of the shift map σ and the metric $\bar{d}$ can be found in [16].

Write

$$\bar{G} = \{(g, g, g \cdots): g \in G\}, \tag{1}$$

$G_\infty = \prod_{i=0}^{\infty} G_i$, where $G_i = G, i \geq 0$.

According to [17, 18], $(\bar{X}, \bar{G}, \bar{d})$ is compact metric $\bar{G}$-space. This paper mainly studies dynamical properties of σ in compact metric $\bar{G}$-space.

Theorem 4. *Let (X, d) be compact metric G-space and the metric d be invariant to G. Then, σ has $\bar{G}$-asymptotic average tracking property.*

Proof. Let $\{\bar{y}^i\}_{i\geq 0}(\bar{y}^i = (y_0^i, y_1^i, y_2^i, \cdots))$ be $\bar{G}$-asymptotic average pseudo orbit of σ. Then, for any nonnegative integer $j \geq 0$, there exists $\bar{t}^j \in \bar{G}$ satisfying.

$$\lim_{n\longrightarrow\infty} \frac{1}{n} \sum_{j=0}^{j=n-1} \bar{d}\left(\bar{t}^j \sigma\left(\bar{y}^j\right), \bar{y}^{j+1}\right) = 0, \tag{2}$$

where $\bar{t}^j = (t_0^j, t_1^j, t_2^j, \cdots)$. □

According to [14], we can choose a zero density set J satisfying

$$\lim_{i\longrightarrow\infty, i\notin J} \bar{d}\left(\bar{t}^i \sigma\left(\bar{y}^i\right), \bar{y}^{i+1}\right) = 0. \tag{3}$$

So for any $\varepsilon > 0$, there exists a positive integer $m > 0$ such that $i \geq m$ and $i \notin J$ implies

$$\bar{d}\left(\bar{t}^i \sigma\left(\bar{y}^i\right), \bar{y}^{i+1}\right) < \varepsilon. \tag{4}$$

So, for any $i \geq m$, $i \notin J$, and $k \geq 1$, it follows that

$$\frac{d\left(t_{k-1}^i y_k^i, y_{k-1}^{i+1}\right)}{2^{k-1}} < \varepsilon. \tag{5}$$

Thus, we can obtain the following inequality

$$\begin{aligned} &d\left(t_{k-1}^i y_k^i, y_{k-1}^{i+1}\right) < 2^{k-1}\varepsilon, \\ &d\left(t_{k-2}^{i+1} y_{k-1}^{i+1}, y_{k-2}^{i+2}\right) < 2^{k-2}\varepsilon, \\ &d\left(t_{k-3}^{i+2} y_{k-2}^{i+2}, y_{k-3}^{i+3}\right) < 2^{k-3}\varepsilon, \\ &\cdots, \\ &d\left(t_1^{i+k-2} y_2^{i+k-2}, y_1^{i+k-1}\right) < 2^1\varepsilon, \\ &d\left(t_0^{i+k-1} y_1^{i+k-1}, y_0^{i+k}\right) < 2^0\varepsilon. \end{aligned} \tag{6}$$

Since the metric d is invariant to G, we can have that

$$\begin{aligned} &d\left(t_0^{i+k-1} t_1^{i+k-2} \cdots t_{k-3}^{i+2} t_{k-2}^{i+1} t_{k-1}^i y_k^i, t_0^{i+k-1} t_1^{i+k-2} \cdots t_{k-3}^{i+2} t_{k-2}^{i+1} y_{k-1}^{i+1}\right) < 2^{k-1}\varepsilon, \\ &d\left(t_0^{i+k-1} t_1^{i+k-2} \cdots t_{k-3}^{i+2} t_{k-2}^{i+1} y_{k-1}^{i+1}, t_0^{i+k-1} t_1^{i+k-2} \cdots t_{k-3}^{i+2} y_{k-2}^{i+2}\right) < 2^{k-2}\varepsilon, \\ &d\left(t_0^{i+k-1} t_1^{i+k-2} \cdots t_{k-4}^{i+3} t_{k-3}^{i+2} y_{k-2}^{i+2}, t_0^{i+k-1} t_1^{i+k-2} \cdots t_{k-4}^{i+3} y_{k-3}^{i+3}\right) < 2^{k-3}\varepsilon, \\ &\cdots, \\ &d\left(t_0^{i+k-1} t_1^{i+k-2} y_2^{i+k-2}, t_0^{i+k-1} y_1^{i+k-1}\right) < 2^1\varepsilon, \\ &d\left(t_0^{i+k-1} y_1^{i+k-1}, y_0^{i+k}\right) < 2^0\varepsilon. \end{aligned} \tag{7}$$

Thus, when $i \geq m$ and $i \notin J$ for any positive integer $k \geq 1$, we can get that

$$\begin{aligned} &d\left(t_0^{i+k-1} t_1^{i+k-2} \cdots t_{k-3}^{i+2} t_{k-2}^{i+1} t_{k-1}^i y_k^i, y_0^{i+k}\right) \\ &\quad < d\left(t_0^{i+k-1} t_1^{i+k-2} \cdots t_{k-3}^{i+2} t_{k-2}^{i+1} t_{k-1}^i y_k^i, t_0^{i+k-1} t_1^{i+k-2} \cdots t_{k-3}^{i+2} t_{k-2}^{i+1} y_{k-1}^{i+1}\right) \\ &\quad + d\left(t_0^{i+k-1} t_1^{i+k-2} \cdots t_{k-3}^{i+2} t_{k-2}^{i+1} y_{k-1}^{i+1}, t_0^{i+k-1} t_1^{i+k-2} \cdots t_{k-3}^{i+2} y_{k-2}^{i+2}\right) \\ &\quad + d\left(t_0^{i+k-1} t_1^{i+k-2} \cdots t_{k-4}^{i+3} t_{k-3}^{i+2} y_{k-2}^{i+2}, t_0^{i+k-1} t_1^{i+k-2} \cdots t_{k-4}^{i+3} y_{k-3}^{i+3}\right) \\ &\quad + \cdots\cdots + d\left(t_0^{i+k-1} t_1^{i+k-2} y_2^{i+k-2}, t_0^{i+k-1} y_1^{i+k-1}\right) \\ &\quad + d\left(t_0^{i+k-1} y_1^{i+k-1}, y_0^{i+k}\right) + \\ &\quad < \left(2^{k-1} + 2^{k-2} + \cdots + 2^0\right)\varepsilon < 2^k\varepsilon. \end{aligned} \tag{8}$$

That is

$$\frac{d\left(t_0^{i+k-1} t_1^{i+k-2} \cdots t_{k-3}^{i+2} t_{k-2}^{i+1} t_{k-1}^i y_k^i, y_0^{i+k}\right)}{2^k} < \varepsilon. \tag{9}$$

Let $\bar{y} = (y_0^0, y_0^1, y_0^2, \cdots) \in \bar{X}$ and $\bar{g}^i = (e, t_0^i, t_0^{i+1} t_1^i, t_0^{i+2} t_1^{i+1} t_2^i, \cdots)$, where $i \geq m$. Then, for any $i \geq m$ and $i \notin J$, it follows that

$$\bar{d}\left(\sigma^i(\bar{y}), \bar{g}^i \bar{y}^i\right) < \varepsilon. \tag{10}$$

Hence,

$$\lim_{i\longrightarrow\infty, i\notin J} \bar{d}\left(\sigma^i(\bar{y}), \bar{g}^i \bar{y}^i\right) = 0. \tag{11}$$

According to [14], we have that

$$\lim_{n\longrightarrow\infty} \frac{1}{n} \sum_{i=0}^{i=n-1} \bar{d}\left(\sigma(\bar{y}), \bar{g}^i \bar{y}^i\right) = 0.. \tag{12}$$

Hence, σ has $\bar{G}$-asymptotic average tracking property.

3. $\bar{G}$-Lipschitz Tracking Property of σ

We can find the definition of G-Lipschitz tracking property in [15].

Theorem 5. *Let (X, d) be compact metric G-space and the metric d be invariant to G. Then, σ has $\bar{G}$-Lipschitz tracking property.*

Proof. Let $L = 2$ and $\varepsilon_0 = 1$. Let $\{\bar{y}^i\}_{i\geq 0}(\bar{y}^i = (y_0^i, y_1^i, y_2^i, \cdots))$ be $(\bar{G}, \varepsilon)$- pseudo orbit of σ for any $0 < \varepsilon < \varepsilon_0$. Then for any $i \geq 0$, there exists $\bar{t}^i \in \bar{G}$ satisfying.

$$\bar{d}\left(\bar{t}^i\sigma\left(\bar{y}^i\right), \bar{y}^{i+1}\right) < \varepsilon, \tag{13}$$

where $\bar{t}^i = (t_0^i, t_1^i, t_2^i, \cdots)$. □

So, when $k \geq 1$ and $i \geq 0$, we can get that

$$\frac{d\left(t_{k-1}^i y_k^i, y_{k-1}^{i+1}\right)}{2^{k-1}} < \varepsilon. \tag{14}$$

Thus, we can obtain the following inequality

$$\begin{gathered} d\left(t_{k-1}^i y_k^i, y_{k-1}^{i+1}\right) < 2^{k-1}\varepsilon, \\ d\left(t_{k-2}^{i+1} y_{k-1}^{i+1}, y_{k-2}^{i+2}\right) < 2^{k-2}\varepsilon, \\ d\left(t_{k-3}^{i+2} y_{k-2}^{i+2}, y_{k-3}^{i+3}\right) < 2^{k-3}\varepsilon, \\ \cdots, \\ d\left(t_1^{i+k-2} y_2^{i+k-2}, y_1^{i+k-1}\right) < 2^1\varepsilon, \\ d\left(t_0^{i+k-1} y_1^{i+k-1}, y_0^{i+k}\right) < 2^0\varepsilon. \end{gathered} \tag{15}$$

Since d is invariant to G, we can have that

$$\begin{gathered} d\left(t_0^{i+k-1} t_1^{i+k-2} \cdots t_{k-3}^{i+2} t_{k-2}^{i+1} t_{k-1}^i y_k^i, t_0^{i+k-1} t_1^{i+k-2} \cdots t_{k-3}^{i+2} t_{k-2}^{i+1} y_{k-1}^{i+1}\right) < 2^{k-1}\varepsilon, \\ d\left(t_0^{i+k-1} t_1^{i+k-2} \cdots t_{k-3}^{i+2} t_{k-2}^{i+1} y_{k-1}^{i+1}, t_0^{i+k-1} t_1^{i+k-2} \cdots t_{k-3}^{i+2} y_{k-2}^{i+2}\right) < 2^{k-2}\varepsilon, \\ d\left(t_0^{i+k-1} t_1^{i+k-2} \cdots t_{k-4}^{i+3} t_{k-3}^{i+2} y_{k-2}^{i+2}, t_0^{i+k-1} t_1^{i+k-2} \cdots t_{k-4}^{i+3} y_{k-3}^{i+3}\right) < 2^{k-3}\varepsilon, \\ \cdots, \\ d\left(t_0^{i+k-1} t_1^{i+k-2} y_2^{i+k-2}, t_0^{i+k-1} y_1^{i+k-1}\right) < 2^1\varepsilon, \\ d\left(t_0^{i+k-1} y_1^{i+k-1}, y_0^{i+k}\right) < 2^0\varepsilon. \end{gathered} \tag{16}$$

Thus, when $k \geq 1$ and $i \geq 0$, we can get that

$$\begin{aligned} & d\left(t_0^{i+k-1} t_1^{i+k-2} \cdots t_{k-3}^{i+2} t_{k-2}^{i+1} t_{k-1}^i y_k^i, y_0^{i+k}\right) \\ & \quad < d\left(t_0^{i+k-1} t_1^{i+k-2} \cdots t_{k-3}^{i+2} t_{k-2}^{i+1} t_{k-1}^i y_k^i, t_0^{i+k-1} t_1^{i+k-2} \cdots t_{k-3}^{i+2} t_{k-2}^{i+1} y_{k-1}^{i+1}\right) \\ & \qquad + d\left(t_0^{i+k-1} t_1^{i+k-2} \cdots t_{k-3}^{i+2} t_{k-2}^{i+1} y_{k-1}^{i+1}, t_0^{i+k-1} t_1^{i+k-2} \cdots t_{k-3}^{i+2} y_{k-2}^{i+2}\right) \\ & \qquad + d\left(t_0^{i+k-1} t_1^{i+k-2} \cdots t_{k-4}^{i+3} t_{k-3}^{i+2} y_{k-2}^{i+2}, t_0^{i+k-1} t_1^{i+k-2} \cdots t_{k-4}^{i+3} y_{k-3}^{i+3}\right) \\ & \qquad + \cdots\cdots + d\left(t_0^{i+k-1} t_1^{i+k-2} y_2^{i+k-2}, t_0^{i+k-1} y_1^{i+k-1}\right) + d\left(t_0^{i+k-1} y_1^{i+k-1}, y_0^{i+k}\right) \\ & \quad < \left(2^{k-1} + 2^{k-2} + \cdots + 2^0\right)\varepsilon < 2^k\varepsilon. \end{aligned} \tag{17}$$

That is,

$$\frac{d\left(t_0^{i+k-1} t_1^{i+k-2} \cdots t_{k-3}^{i+2} t_{k-2}^{i+1} t_{k-1}^i y_k^i, y_0^{i+k}\right)}{2^k} < \varepsilon. \tag{18}$$

Let

$$\begin{gathered} \bar{y} = \left(y_0^0, y_0^1, y_0^2, \cdots\right) \in \bar{X}, \\ \bar{g}^i = \left(e, t_0^i, t_0^{i+1} t_1^i, t_0^{i+2} t_1^{i+1} t_2^i, \cdots\right). \end{gathered} \tag{19}$$

Hence, we have that

$$\bar{d}\left(\sigma^i(\bar{y}), \bar{g}^i\bar{y}^i\right) < \varepsilon < L\varepsilon. \tag{20}$$

So, σ has $\bar{G}$-Lipschitz tracking property.

4. $\bar{G}$-Periodic Tracking Property of σ

We can find the concepts of G-periodic point and G-periodic tracking property in [19, 20].

Theorem 6. *Let (X, d) be compact metric G-space and the metric d be invariant to G. Then, σ has $\bar{G}$-periodic tracking property.*

Proof. Let $0 < \delta < \varepsilon$ for any $\varepsilon > 0$ and $\{\bar{y}^i\}_{i\geq 0}(\bar{y}^i = (y_0^i, y_1^i, y_2^i, \cdots))$ be $(\bar{G}, \delta)$- pseudo orbit of σ. Thus, there exists $n > 0$ satisfying.

$$\bar{y}^{kn+j} = \bar{y}_j, 0 < k, 0 \leq j < n. \tag{21}$$

□

Write $\bar{y} = (y_0^0, y_0^1, y_0^2, \cdots) \in \bar{X}$. It follows that

$$\bar{e}\sigma^n(\bar{y}) = \bar{y}. \tag{22}$$

Hence, $\bar{y} \in P_{\bar{G}}(\sigma)$. In addition, for any $i \geq 0$, there exists $\bar{t}^i \in \bar{G}$ satisfying

$$\bar{d}\left(\bar{t}^i\sigma(\bar{y}^i), \bar{y}^{i+1}\right) < \delta, \tag{23}$$

where $\bar{t}^i = (t_0^i, t_1^i, t_2^i, \cdots)$.

So, when $k \geq 1$ and $i \geq 0$, we can get that

$$\frac{d\left(t_{k-1}^i y_k^i, y_{k-1}^{i+1}\right)}{2^{k-1}} < \delta. \tag{24}$$

Thus, we can obtain the following inequality

$$\begin{gathered} d\left(t_{k-1}^i y_k^i, y_{k-1}^{i+1}\right) < 2^{k-1}\delta, \\ d\left(t_{k-2}^{i+1} y_{k-1}^{i+1}, y_{k-2}^{i+2}\right) < 2^{k-2}\delta, \\ d\left(t_{k-3}^{i+2} y_{k-2}^{i+2}, y_{k-3}^{i+3}\right) < 2^{k-3}\delta, \\ \cdots, \\ d\left(t_1^{i+k-2} y_2^{i+k-2}, y_1^{i+k-1}\right) < 2^1\delta, \\ d\left(t_0^{i+k-1} y_1^{i+k-1}, y_0^{i+k}\right) < 2^0\delta, \end{gathered} \tag{25}$$

Since d is invariant to G, we can have that

$$\begin{gathered} d\left(t_0^{i+k-1} t_1^{i+k-2} \cdots t_{k-3}^{i+2} t_{k-2}^{i+1} t_{k-1}^i y_k^i, t_0^{i+k-1} t_1^{i+k-2} \cdots t_{k-3}^{i+2} t_{k-2}^{i+1} y_{k-1}^{i+1}\right) < 2^{k-1}\delta, \\ d\left(t_0^{i+k-1} t_1^{i+k-2} \cdots t_{k-3}^{i+2} t_{k-2}^{i+1} y_{k-1}^{i+1}, t_0^{i+k-1} t_1^{i+k-2} \cdots t_{k-3}^{i+2} y_{k-2}^{i+2}\right) < 2^{k-2}\delta, \\ d\left(t_0^{i+k-1} t_1^{i+k-2} \cdots t_{k-4}^{i+3} t_{k-3}^{i+2} y_{k-2}^{i+2}, t_0^{i+k-1} t_1^{i+k-2} \cdots t_{k-4}^{i+3} y_{k-3}^{i+3}\right) < 2^{k-3}\delta, \\ \cdots, \\ d\left(t_0^{i+k-1} t_1^{i+k-2} y_2^{i+k-2}, t_0^{i+k-1} y_1^{i+k-1}\right) < 2^1\delta, \\ d\left(t_0^{i+k-1} y_1^{i+k-1}, y_0^{i+k}\right) < 2^0\delta, \end{gathered} \tag{26}$$

Thus, when $k \geq 1$ and $i \geq 0$, we can get that

$$\begin{aligned} & d\left(t_0^{i+k-1} t_1^{i+k-2} \cdots t_{k-3}^{i+2} t_{k-2}^{i+1} t_{k-1}^i y_k^i, y_0^{i+k}\right) \\ & \quad < d\left(t_0^{i+k-1} t_1^{i+k-2} \cdots t_{k-3}^{i+2} t_{k-2}^{i+1} t_{k-1}^i y_k^i, t_0^{i+k-1} t_1^{i+k-2} \cdots t_{k-3}^{i+2} t_{k-2}^{i+1} y_{k-1}^{i+1}\right) \\ & \qquad + d\left(t_0^{i+k-1} t_1^{i+k-2} \cdots t_{k-3}^{i+2} t_{k-2}^{i+1} y_{k-1}^{i+1}, t_0^{i+k-1} t_1^{i+k-2} \cdots t_{k-3}^{i+2} y_{k-2}^{i+2}\right) \\ & \qquad + d\left(t_0^{i+k-1} t_1^{i+k-2} \cdots t_{k-4}^{i+3} t_{k-3}^{i+2} y_{k-2}^{i+2}, t_0^{i+k-1} t_1^{i+k-2} \cdots t_{k-4}^{i+3} y_{k-3}^{i+3}\right) \\ & \qquad + \cdots\cdots + d\left(t_0^{i+k-1} t_1^{i+k-2} y_2^{i+k-2}, t_0^{i+k-1} y_1^{i+k-1}\right) + d\left(t_0^{i+k-1} y_1^{i+k-1}, y_0^{i+k}\right) \\ & \quad < \left(2^{k-1} + 2^{k-2} + \cdots + 2^0\right)\delta < 2^k\delta. \end{aligned} \tag{27}$$

That is,

$$\frac{d\left(t_0^{i+k-1} t_1^{i+k-2} \cdots t_{k-3}^{i+2} t_{k-2}^{i+1} t_{k-1}^i y_k^i, y_0^{i+k}\right)}{2^k} < \delta. \tag{28}$$

Let $\bar{g}^i = (e, t_0^i, t_0^{i+1} t_1^i, t_0^{i+2} t_1^{i+1} t_2^i, \cdots)$ for any $i \geq 0$. Hence, we have that

$$\bar{d}\left(\sigma^i(\bar{y}), \bar{g}^i \bar{y}^i\right) < \delta < \varepsilon. \tag{29}$$

So, the shift map σ has $\bar{G}$-periodic tracking property.

5. Conclusion

Firstly, we introduced the concept of$G$-Lipschitz tracking property, G-asymptotic average tracking property, and G-periodic tracking property. Secondly, we studied their dynamical properties and topological structure in the infinite product space under group action and obtained the following conclusions: (1) let (X, d) be compact metric G – space and the metric d be invariant to G. Then, σ has $\bar{G}$-asymptotic average tracking property; (2) let (X, d) be compact metric G-space and the metric d be invariant to G. Then, σ has $\bar{G}$-Lipschitz tracking property; (3) let (X, d) be compact metric G-space and the metric d be invariant to G. Then, σ has $\bar{G}$-periodic tracking property. The above results make up for the lack of theory of G-Lipschitz tracking property, G-asymptotic average tracking property, and G-periodic tracking property in infinite product space under group action.

Data Availability

The data used to support the findings of this study are included within references [1–20] in the article.

Conflicts of Interest

The author declares that he has no conflicts of interest.

Acknowledgments

Research was partially supported by the NSF of Guangxi Province (2020JJA110021) and the construction project of Wuzhou University of China (2020B007).

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